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Nonlinear Control of Flat Systems Using a Non-Flat Output with Dynamic Extension

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***Abstract*—If a nonlinear system is differentially flat and a flat output is known, the design of a linearizing feedback law is straightforward. For state-space systems, this corresponds to the input-to-state linearization. Otherwise, i.e., if the system is not flat or no flat output can be found, we could carry out an input-output linearization provided the system is minimum phase. In this case, only certain parts of the systems dynamics are assigned by the control law. From a theoretical point of view, this method is based on the Byrnes-Isidori normal form. A less common approach is the usage of the non-flat output in order to carry out a linearization in connection with the generalized controller canonical form [8]. The linearization is achieved by a dynamic extension. The existence of an alternative linearization method may be advantageous from a computational point of view and gives additional degrees of freedom, e.g. allowing for a higher-order of the desired closed-loop dynamics. Both approaches are illustrated on the nonlinear boost converter model.**

***Index Terms*—Flat systems, non-flat output, zero dynamics, minimum phase, dynamic extension**

I. Introduction

The concept of differential flatness introduced by Fliess et al. [9] more than twenty years ago had a great impact on control theory. If a flat output is known, controller design can be simplified significantly. Unfortunately, deciding whether a system is flat or computing the flat output are often difficult tasks. Although many researches worked on this problem and its mathematical background as well as on the implementation of appropriate software packages [1], [2], [10]–[13], [26], [35], [36], no general solution has been found.

Another approach to nonlinear control is exact feedback linearization [21]. A single-input single-output system is flat if and only if it is exactly input-to-state linearizable [42]. The situation is more complicated for multiple-input multiple-output systems [25]. The relation between a flat and a non-flat output is investigated in [18].

If a system has a well-defined relative degree, one can design an input-output linearizing feedback law provided the system is minimum phase [22]. If the relative degree is strictly less then the dimension of the state-space, this approach imposes new dynamics only on a submanifold of the state-space. In contrast, applying dynamic extension [8] allows to generate new linear dynamics on the full state-space. Moreover, an explicit input-to-state linearization is avoided as the control law is derived from a non-flat output.

The paper is structured as follows. In Section II we recall some control theoretic preliminaries. In Section III we discuss the controller design by dynamic extension for minimum phase systems. Further research to extend this approach to a broader class of systems is discussed in Section V. Finally, we draw some conclusions in Section VI.

II. Preliminaries

A. Relative Degree and Exact Input-Output Linearisation

Consider a nonlinear input affine system

$$\dot{x} = f(x) + g(x)u, \quad y = h(x) \tag{1}$$

with vector fields $f, g : \mathcal{M} \to \mathbb{R}^n$ and a scalar field $h : \mathcal{M} \to \mathbb{R}$ defined on an open subset $\mathcal{M} \subseteq \mathbb{R}^n$. We assume that all maps are sufficiently smooth. The *Lie derivative* of the scalar field h along the vector field f is defined by $L_f h(x) := h'(x)f(x)$. System (1) has a *relative degree* r at a point $x^0 \in \mathcal{M}$ if $L_g h(x) = L_g L_f h(x) = \cdots = L_f g L_f^{r-1} h(x) = 0$ for all x in a neighborhood of x^0 and $L_g L_f^{r-1} h(x^0) \neq 0$. The single-input single-output system (1) is *flat* if and only if there exists an output with relative degree $r = n$, see [42].

Now, we assume that system (1) is flat but has a well-defined relative degree $r < n$ w.r.t. the output y, i.e., y is a non-flat output. Then we have

$$\begin{array}{rcccl} y & = & \phi_1(x) & = & h(x), \\ \dot{y} & = & \phi_2(x) & = & L_f h(x), \\ \vdots & & & \vdots & \\ y^{(r-1)} & = & \phi_r(x) & = & L_f^{r-1} h(x). \end{array} \tag{2}$$

Due to the relative degree, the time derivative of order r depends explicitly on the input u:

$$y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} h(x) u. \tag{3}$$

Using this equation we can linearize the input-output behavior of the system setting

$$L_f^r h(x) + L_g L_f^{r-1} h(x) u \overset{!}{=} v$$

with the virtual input v. The associated control law is given by

$$u = \frac{1}{L_g L_f^{r-1} h(x)} \left(v - L_f^r h(x) \right). \tag{4}$$

This results in linear dynamics

$$y^{(r)} = v \tag{5}$$

in form of a chain of r integrators. This linear system can be stabilized by a state feedback

$$v = -k_0 z_1 - \cdots - k_{r-1} z_r + k_0 w \tag{6}$$

with coefficients $k_0, \ldots, k_{r-1}$ and the reference variable w. The associated control law reads

$$u = \frac{1}{L_g L_f^{r-1} h(x)} \left(k_0 w - \sum_{i=0}^{r} k_i L_f^i h(x) \right) \tag{7}$$

with $k_r := 1$. Replacing the components of the state by the output and its time derivatives as in (2) we can write the resulting system as

$$y^{(r)} + k_{r-1} y^{(r-1)} + \cdots + k_0 y = k_0 w. \tag{8}$$

B. Byrnes-Isidori Normal Form

To investigate the remaining dynamics of the system, the coordinates $\phi_1, \ldots, \phi_r$ from (2) are augmented by $n - r$ additional functions $\phi_{r-1}, \ldots, \phi_n$ such that

$$L_g \phi_i(x) \equiv 0 \quad \text{for} \quad i = n+1, \ldots, n. \tag{9}$$

The resulting change of coordinates

$$z = \Phi(x) \tag{10}$$

transforms system (1) into the *Byrnes-Isidori normal form* [21], [22]

$$\begin{array}{rcl} \dot{z}_1 &=& z_2 \\ &\vdots& \\ \dot{z}_{r-1} &=& z_r \\ \dot{z}_r &=& \alpha^*(z) + \beta^*(z) u \\ \dot{z}_{r+1} &=& q_1(z) \\ &\vdots& \\ \dot{z}_n &=& q_{n-r}(z) \\ y &=& z_1. \end{array} \tag{11}$$

with

$$\begin{array}{rcl} \alpha^*(z) &=& L_f^r h(\Phi^{-1}(z)) \\ \beta^*(z) &=& L_g L_f^{r-1} h(\Phi^{-1}(z)). \end{array} \tag{12}$$

Under this transformation, the system is decomposed into two subsystems. The first subsystem is defined by the coordinates $z_1, \ldots, z_r$, whereas the second subsystem by $z_{r+1}, \ldots, z_n$.

System (1) is linearized by the feedback (4) corresponding to

$$u = \frac{1}{\beta^*(z)} \left(v - \alpha^*(z) \right) \tag{13}$$

in the transformed coordinates. The second subsystem does not directly depend on the input u due to (9). With (13), the dynamics of the first subsystem are decoupled from the second subsystem. In order to achieve stability of the whole controlled system, we assume that the second subsystem is asymptotically stable for $z_1 = \cdots = z_r = 0$. The resulting dynamics is called *zero dynamics*. A system with asymptotically stable equilibrium of the zero dynamics is called *minimum phase*.

Note that the symbolic computation of the Byrnes-Isidori normal form or the zero dynamics can be challenging [5], [31]. In these cases, one could try of investigate the stability using a Taylor linearization of system (1) in the original coordinates. Such a equilibrium point is called *hyperbolic* if no eigenvalue of the associated Jacobian lies on the imaginary axis. Then, the linearized system is asymptotically stable iff the equilibrium point of the original nonlinear system is asymptotically stable due to the Theorem of Hartman and Grobman [3], [17].

C. Generalized Controller Canonical Form

Now, we will construct a different frame of coordinates for the second subsystem. We start with the output time derivative (3), which depends on u. The next time derivative additionally depends on $\dot{u}$ and is more complicated

$$\begin{aligned} y^{(r+1)} =& L_f^{r+1} h(x) + L_g L_f^r h(x) u + L_f L_g L_f^{r-1} h(x) u \\ &+ L_g^2 L_f^{r-1} h(x) u^2 + L_g L_f^{r-1} h(x) \dot{u}, \end{aligned} \tag{14}$$

see [24]. We use the time derivatives

$$\begin{array}{rcl} y^{(r)} &=& \phi_r(x, u) \\ y^{(r+1)} &=& \phi_{r+1}(x, u, \dot{u}) \\ &\vdots& \\ y^{(n)} &=& \phi_{r+1}(x, u, \dot{u}, \ddot{u}, \ldots, u^{(n-r+1)}) \end{array} \tag{15}$$

to augment the maps (2) into a change of coordinates

$$z = \Phi(x, u, \dot{u}, , \ddot{u}, \ldots, u^{(n-r+1)}) \tag{16}$$

transforming (1) into the *generalized controller canonical form (GCCF)* [8]:

$$\begin{array}{rcl} \dot{z}_1 &=& z_2 \\ &\vdots& \\ \dot{z}_{n-1} &=& z_n \\ \dot{z}_n &=& \gamma(z, u, \dot{u}, \ddot{u}, \ldots, u^{(n-r)}). \end{array} \tag{17}$$

Since system (1) is affine in input u, the map γ is affine in the highest order time derivative $u^{(n-r)}$, i.e.,

$$\begin{aligned} &\gamma(z, u, \dot{u}, \ldots, u^{(n-r)}) = \\ &\alpha(z, u, \ldots, u^{(n-r-1)}) + \beta(z, u, \ldots, u^{(n-r-1)}) u^{(n-r)}. \end{aligned} \tag{18}$$

Remark 1: Because the right hand side of system (17) depends explicitly on the time derivatives $\dot{u}, \ddot{u}, \ldots, u^{(n-r)}$ of the input u, it is not a classical but a *generalized state-space* *description*. Similarly, the transformation (16) is a *generalized state transformation*.

Remark 2: The linearization

$$\begin{array}{rclcrcl} a_0 & = & -\frac{\partial\gamma}{\partial z_1}, & \ldots, & a_{n-1} & = & -\frac{\partial\gamma}{\partial z_n} \\ b_0 & = & \frac{\partial\gamma}{\partial u}, & \ldots, & b_{n-r} & = & \frac{\partial\gamma}{\partial u^{(n-r)}} \end{array}$$

of system (17) in a considered operating point yields the differential equation

$$y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_0 y = b_{n-r}u^{(n-r)} + \cdots + b_0 u.$$

The corresponding transfer function has the form

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_{n-r}s^{n-r} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}. \quad (19)$$

If the zero dynamics is hyperbolic, its stability can be deduced from numerator polynomial of (19).

III. Dynamic Extension for Minimum Phase Systems

A. Design Procedure

Consider a nonlinear system (1) with a relative degree $r < n$. The system can be transformed into the GCCF (17). We assume that the considered operating point of the zero dynamics is hyperbolic and asymptotically stable, i.e., the numerator polynomial of (19) is a Hurwitz polynomial.

First, we want to compensate the system's nonlinearities by feedback. In order to meet this goal we set

$$\begin{array}{rcl} \dot{z}_n & = & \gamma(z, u, \dot{u}, \ldots, u^{(n-r)}) \\ & = & \alpha(z, u, \ldots) + \beta(z, u, \ldots)u^{(n-r)} \\ & \stackrel{!}{=} & v \end{array} \quad (20)$$

with the virtual input v. Due to the structure (18) of the map γ, we can resolve (20) w.r.t. the highest order time derivative of the input signal:

$$u^{(n-r)} = \frac{v - \alpha(z, u, \ldots)}{\beta(z, u, \ldots)}. \quad (21)$$

Applying (21) to (20) results in a chain of integrators

$$y^{(n)} = v \quad (22)$$

similar to (5) but of length n.

Next, we want to steer the system to the dynamics

$$y^{(n)} + k_{n-1}y^{(n-1)} + \cdots + k_0 y = k_0 w$$

of a linear system with the reference variable w, where $k_0, \ldots, k_{n-1}$ are the coefficients of the desired characteristic polynomial

$$s^n + k_{n-1}s^{n-1} + \cdots + k_1 s + k_0. \quad (23)$$

This linear dynamics can be imposed into (22) by the feedback

$$v = -k_0 z_1 - \cdots - k_{n-1}z_n + k_0 w \quad (24)$$

as in (6). Combining the linearizing feedback (21) with (24) yields

$$u^{(n-r)} = -\frac{\alpha(z, u, \ldots) + k_0 z_1 + \cdots + k_{n-1}z_n - k_0 w}{\beta(z, u, \ldots)}. \quad (25)$$

We can express this control law in original coordinates using the inverse transformation of (16). The structure of the resulting controller is shown in Fig. 1. The controller does not only feed back the state, but also contains a chain of integrators to generate u from $u^{(n-r)}$. The dynamics w.r.t. u corresponds to the zero dynamics which is required to be asymptotically stable.

B. Implications and Advantages

The alternative approach to feedback linearization has some interesting properties for the computation:

Remark 3: In flatness based control, the GCCF is also used for *quasi-static state feedback* [6], [33], [34] of multiple-input multiple-output systems. For quasi-static state feedback, the input-output relations are solved w.r.t. the input signals, i.e., for the lowest order time derivatives of the inputs. This way one avoids a dynamic extension, see [21, Sect. 5.4]. The approach discussed here is somehow reciprocal. It was mentioned in [8] but rarely used in practice. This way, we don't explicitly know the solution of (9), which is formally a partial differential equation [5], [31].

Remark 4: The symbolic computation of a flat output requires sophisticated methods from computer algebra [1], [13]. Dynamic extension avoids this obstacle. As a matter of fact, symbolic computations can be omitted. The time derivatives of the output required to construct the control law (25) in original coordinates can be calculated using alternative differentiation technique called *automatic* or *algorithmic differentiation*, see [16], [32].

Several design methods are directly based on appropriate normal forms. Quite often, the explicit computation of the inverse transformations may be difficult. With the dynamic extension one has a further transformation, which may be advantageous:

Remark 5: For example, the observer suggested in [23] extends the high gain observer form [14] to the first subsystem of the Byrnes-Isidori normal form (11). Although the observer gain can be implemented in the original coordinates, the transformation is still required. If solving (9) is avoided using an input-output form as in [29], it becomes much more difficult to show the convergence of the observer. In a manner similar to [14] one could also design a high gain observer using GCCF (17). The occurrence of time derivatives of the input is not a problem since these signals are provided by the controller (25). Contrary ot [23], [29], where a detectability condition concerning the second subsystem is required, we can impose full order dynamics to the observer.

Remark 6: Internal model control (IMC) is widely used in process engineering [19]. In case of nonlinear systems, the controller can easily be computed for flat systems if the flat output is known or for minimum phase (possibly no-flat) input-output linearizable systems [38], [39]. The dynamic extension can be adapted to these methods, where the inversion based on (4) and (5) in the sense of a Hirschorn inverse [20] is replaced by the dynamic inversion (21) and (22). This results in an alternative control law.

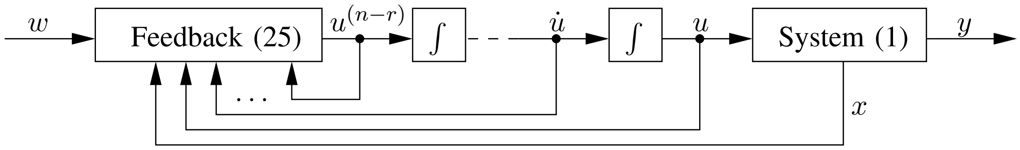

Fig. 1. Structure of the dynamic controller (25) with system (1)

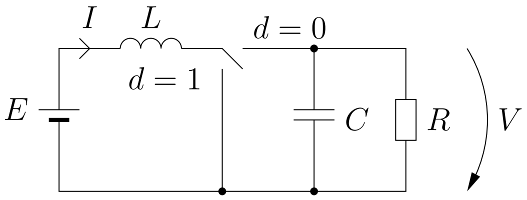

Fig. 2. Network model of the boost converter

IV. Example

A *boost converter* or *step-up converter* is a DC to DC converter with an output voltage V greater than the source voltage E. The network model of a boost converter with load resistor R is sketched in Fig. 2. The converter consists of two reactive network elements, an inductor L and a capacitor C. The switch is typically a BJT, MOSFET, or IGBT.

The network equations are derived from Kirchhoff's circuit laws with the inductor current I. Using pulse-width modulation (PWM), the discrete switching signal $d(t) \in \{0, 1\}$ can be modeled as a continuous input signal $u \in [0, 1]$ called *supply rate*. We obtain the averaged model

$$\begin{aligned} \dot{x}_1 &= -(1-u)\tfrac{1}{L}x_2 + \tfrac{E}{L}, \\ \dot{x}_2 &= (1-u)\tfrac{1}{C}x_1 - \tfrac{1}{RC}x_2 \end{aligned} \tag{26}$$

with the state vector $x = (x_1, x_2)^T = (I, V)^T$. We take the parameter values from [4, Sect. 8.6.1]: $E = 15\,\mathrm{V}$, $L = 0.5\,\mathrm{mH}$, $C = 1000\,\mu\mathrm{F}$, $R = 10\,\Omega$. We consider the equilibrium point $u^0 = 0.4$, $x_1^0 = 25/6\,\mathrm{A} = 4.1\bar{6}\,\mathrm{A}$, $x_2^0 = 25\,\mathrm{V}$. The linearized system with the output (28) has the transfer function

$$G(s) = \frac{50000(s + 200)}{s^2 + 100s + 720000} \tag{27}$$

The roots $s_{1,2} = -50 \pm 50\sqrt{287}$ of the denominator polynomial are the eigenvalues of the open-loop system. Since these eigenvalues lie in the complex left half-plane, the equilibrium point is hyperbolic and asymptotically stable. In the time domain, this complex conjugate pair of eigenvalues corresponds to declining oscillations.

A. Input-Output Linearization

The feedback linearization of this system has been investigated in many publications [7], [15], [27], [40], [41]. We use the inductor current as an output

$$y = x_1. \tag{28}$$

The time derivative

$$\dot{y} = \dot{x}_1 = \frac{(u-1)x_2 + E}{L} = \underbrace{\frac{E - x_2}{L}}_{L_f h(x)} + \underbrace{\frac{x_2}{L}}_{L_g h(x)} u \tag{29}$$

depends explicitly on the input u. We have $L_g h(x) \neq 0$ for $x_2 \neq 0$, i.e., the system has relative degree $r = 1$ if $x_2 \neq 0$. The linearizing and stabilizing control law (8) reads

$$\begin{aligned} u &= \tfrac{1}{L_g h(x)}\left(k_0(w - h(x) - L_f h(x)\right) \\ &= \tfrac{x_2 - E - k_0 L(x_1 - w)}{x_2} \end{aligned} \tag{30}$$

with the coefficient $k_0 > 0$.

B. Byrnes-Isidori Normal Form

The zero dynamics of this system have been computed symbolically in [27], [40]. To verify these results, we consider the transformation (10) given by

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= Lx_1^2 + Cx_2^2. \end{aligned} \tag{31}$$

The inverse transformation $x = \Phi^{-1}(x)$ has the form

$$\begin{aligned} x_1 &= z_1 \\ x_2 &= \sqrt{\tfrac{z_2 - Lz_1^2}{L}}. \end{aligned} \tag{32}$$

The nonlinearities (12) of the first subsystem are calculated as

$$\begin{aligned} \alpha^*(z) &= \tfrac{1}{C}\sqrt{\tfrac{z_2 - Lz_1^2}{L}} - \tfrac{z_1}{RC} \\ \beta^*(z) &= -\tfrac{1}{C}\sqrt{\tfrac{z_2 - Lz_1^2}{L}}. \end{aligned} \tag{33}$$

Note that the transformation (32) and the nonlinearities (33) are not defined for $z_2 < Lz_1^2$.

C. Zero Dynamics

From the transformations (31) and (32) we calculate the internal dynamics, i.e., the dynamics of the second subsystem of the Byrnes-Isidori normal form:

$$\dot{z}_2 = 2Ez_1 - 2\frac{z_2 - Lz_1^2}{RC}. \tag{34}$$

The zero dyanmics is obtained setting $z_1 = 0$:

$$\dot{z}_2 = -2\frac{z_2}{RC}. \tag{35}$$

The zero dynamics (35) is asymptotically stable, i.e., the system is minimum phase. We verify this result using the transfer function (27). The numerator polynomial has a single root at $s = -200$. Therefore, the zero dynamics is hyperbolic and asymptotically stable. Hence, system (26) with the output (28) is minimum phase.

D. Linearization by Dynamic Extension

Now, we transform system (26) into GCCF (17). From (28) and (29) we obtain the change of coordinates (16) as

$$z = \Phi(x,u) = \begin{pmatrix} x_1 \\ \frac{(u-1)x_2+E}{L} \end{pmatrix} \tag{36}$$

with the inverse

$$x = \Phi^{-1}(x,u) = \begin{pmatrix} z_1 \\ \frac{Lz_2-E}{u-1} \end{pmatrix}. \tag{37}$$

Applying this transformation to system (26) results in the GCCF

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\tfrac{z_2+R(u-1)^2-E}{CLR} + \tfrac{Lz_2-E}{L(u-1)}\dot{u} \\ y &= z_1. \end{aligned} \tag{38}$$

Note that the transformations (36) and (37) as well as the system (38) are defined anywhere except for $u = 1$.

The desired dynamics are described by a characteristic polynomial (23) with $n = 2$. The polynomial is Hurwitz iff $k_0, k_1 > 0$. The feedback law (25) reads

$$\begin{aligned} \dot{u} &= -\tfrac{(u-1)(L(CRk_1-1)z_2-R(u^2-2u-CLk_0+1)z_1-CLRk_0w+E)}{CR(Lz_2-E)} \\ &= -\tfrac{(CRk_1-1)(u-1)x_2-R(u^2-2u-CLk_0+1)x_1-CLRk_0w+CERk_1}{CRx_2} \end{aligned} \tag{39}$$

where the transformed coordinates are replaced by the original coordinates using (36).

E. Simulation Results

We want to steer the system to an output voltage of 25 V starting from the initial values $x_1(0) = 2.\bar{6}\,\mathrm{A}$ and $x_2(0) = 20\,\mathrm{V}$ as in [30]. The open-loop system (26) is simulated with the input $u^0 = 0.4$. For the control laws (30) and (39) we used the reference current $w = 4.1\bar{6}\,\mathrm{A}$. For the input-output linearization by (30) we assign a single eigenvalue $s_1 = -300$, i.e., $k_0 = 300$. For the control law based on the GCCF (39) we select the first eigenvalue identical to the input-output linearization, i.e. $s_1 = -300$, and the second eigenvalue as $s_2 = -3000$. The corresponding coefficients are $k_0 = 900000$ and $k_1 = 3300$. In case of (39) we use the initial value $u(0) = 0.4$. Fig. 3 shows the simulation results. As can be seen the controller based on the GCCF can achieve a faster convergence.

V. Future work

Applying the described approach, i.e., dynamic extension for a system in GCCF, to a system with an unstable zero dynamics does not result in the desired closed-loop dynamics. This should be clear in the case of linear systems, where this approach resembles the pole/zero cancellations in the right-half plane. In case of a nonlinear system with unstable zero dynamics, the proposed compensation-based control design procedure results in an unstable closed-loop system and thus cannot be applied directly. It is well-known, that the location of system zeros, in case of a linear system, cannot be influenced by feedback. Similarly, feedback cannot be applied to stabilize the zero dynamics and solve the aforementioned problem. However, as has been shown in [37], augmenting the system with a parallel compensator, i.e., a dynamical system in parallel to the investigated process, adds a degree of freedom to influence the zero location and thus the stability of the zero dynamics of the new augmented process. A systematic procedure, which allows to design parallel compensators resulting in stable zero dynamics for linear systems has been presented in [28]. Therefore, future work will be concerned with controller design for non-minimum phase systems. In case of the example system, we want to design a controller directly using the capacitor voltage as controlled output.

VI. Conclusions

We provided an example for a comparatively unusual approach in the field of exact feedback linearization. Contrary to standard approaches, the controller design is not based on the Byrnes-Isidori normal form. Our control law is based on a dynamic extension. Different to [21, Chapter 5], the dynamic extension is not used to achieve (otherwise not well-defined) relative degree. We avoid the difficulty to compute a flat output. Future work will be concerned with the extension to multiple-input multiple-output systems as well as to the non-minimum phase systems.

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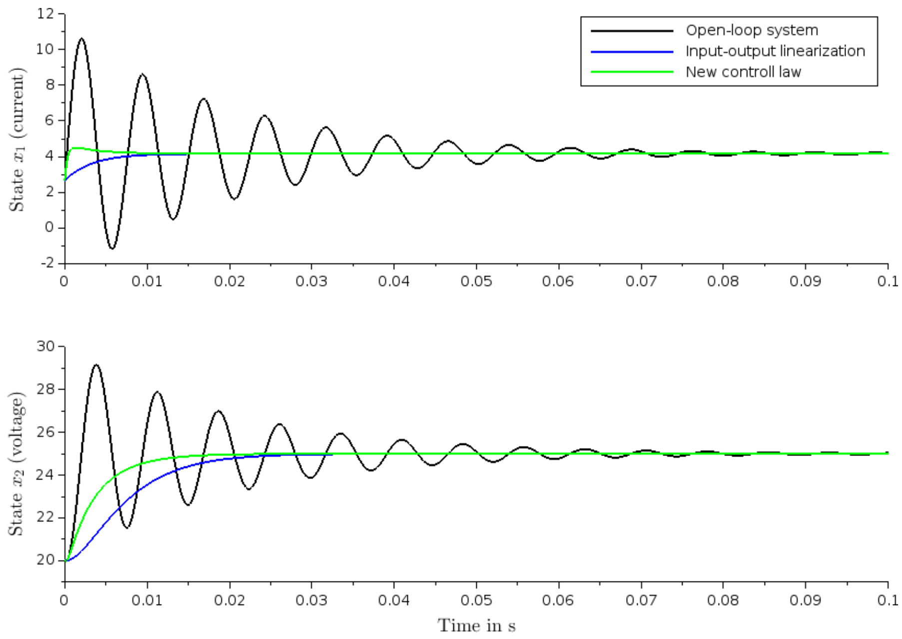

Fig. 3. Simulation results of the boost converter

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